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Study of Sunspot Time Series Using Wavelet-based Multifractal Analysis during Solar Cycle 23 and Ascending Phase of Cycle 24

S. K. Kasde[1*], D. K. Sondhiya[2] and A. K. Gwal[3]

[1]Space Science Research Laboratory, Department of Physics and Electronics, Barkatullah University, Bhopal 462 026, India.
[2]Department of Education in Science and Mathematics, Regional Institute of Education, Shyamla Hills, Bhopal 462013, India.
[3]AISECT University, Chiklod Road, Raisen, 464993, India.

Authors' contributions

This work was carried out in collaboration between all authors. Authors SKK and DKS designed the study, performed the statistical analysis, wrote the protocol, wrote the first draft of the manuscript and managed the analyses of the study. Author AKG managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

Wavelet based Multifractal analysis techniques provides a sophisticated statistical characterization of many complex dynamical phenomena related with Sun and its environment. In this work multifractal property of the Sunspot number time series, has been analyzed during Solar cycle 23 and ascending phase of Solar cycle 24 using Wavelet transform and wavelet based multifractal approach. Present analysis has been performed using the software FRACLAB, developed at INRIA and available online at http://www-rocq.inria.fr. It was found that the singularities spectrum for sunspot time series was well Gaussian in shape suggesting the multifractal characteristics of time series. Thus we conclude that the multifractal based approach provide the local and adaptive description of dynamical processes related with Sun and its climate and can be applied effectively in the study of solar activity.

**Corresponding author: E-mail: sbskasde@gmail.com;*

Keywords: Sunspot number; magnetic field; multifractal analysis and wavelet transform technique.

1. INTRODUCTION

The significant feature of the Sun's outer regions is the existence of a reasonably strong magnetic field which governs all solar activities inside the Sun and its atmosphere. According to the lowest order of approximation, the magnetic field of the Sun is dipolar and axisymmetric in nature. In some localized regions known as sunspot the value of magnetic field are much higher [1]. Sunspots are generally seen in pairs or in groups of pairs at both sides of the solar equator. According to Petrovaye [2] as the sunspot cycle progresses, spots appear closer to sun's equator giving rise to the so called "butterfly diagram" in the time latitude distribution. The twisted magnetic fields above sunspots are sites where solar flares are observed. Bray [1] has been found that chromospheric flares show a very close statistical relationship with sunspots. The number of sunspots continuously changing in time in a random fashion and constitutes a typically random time series [3]. The newly corrected sunspot time series [4-7] progressively indicates the declination in solar activity before the commencement of the Maunder Minimum, while the slow rising drifts in activity after the Maunder Minimum. It shows that by the mid 18th century, solar activity returned to levels corresponding to those observed in current solar cycles. Also Gkana and Zachilas [8] analyze Sunspot number version 2.0 data and claim that prolonged solar activity minimum is probable occur, lasting up to the year ≈ 2100.

Analysis of sunspot could lead significant improvement in the measurement of solar activity [9]. Recently sunspots and related activities have been analyzed by various methods, including correlation analysis [10], Chaos analysis [11,12] and multifractal analysis [13-15]. Schatten [16] used SODA index (Solar Dynamo Amplitude) for understanding of the Sun's dynamo processes to explain the connection between how the Sun's fields are generated and how the Sun broadcasts its future activity levels to Earth. Zachilas and Gkana [17] analyze the yearly data of mean sunspot-number during the period of 1700 to 2012 and concluded that the yearly sunspot number is a low-dimensional deterministic chaotic system. Tarbell et al. [18] used the fractal analysis technique in the context of solar magnetic field to find a fractal dimension. Many authors [18-20] used fractal analysis technique to study the photospheric magnetic structure [18,20]. Tao et al. [21] applied numerical simulate distribution for the multifractal analysis of surface magnetic field. Later, numerical simulations of multifractality and intermittency of the solar structure were performed by so many researchers [22-26]. Multifractal theory provides an elegant statistical characterization of many complex dynamical variations in nature and engineering [27,28]. It is conceivable that it may enrich characterization of the sun's magnetic activity and its dynamical modeling [29-30]. The relative fraction of small scale fluctuation in the magnetic field contributes significantly and reach a critical state of intermittency more prior to flaring [26]. It was found that active regions reach a critical state of intermittency prior to flaring [26]. The multifractal scaling behaviors reported by Movahed [31] are valid for timescales up to more than 50 years. McAteer [32] found analytical connection between multifractal formalism and the set of 3D equations that govern the small-scale and large-scale magnetic structure on the Sun [33,34]. Recently, Georgoulis [35] and McAteer [32] achieved a contrary conclusion that studies of multifractals do not provide a predictive ability for the onset of solar flares. In this paper we have used the multifractal techniques and noticed the presence of multifractality in sunspot number time series during the Solar Cycle 23 and ascending phase of Solar Cycle 24.

2. DATA SET

In this analysis we used the monthly counts of sunspot number for the multifractal analysis for the time span of 1996 to 2016. This period includes complete Solar Cycles of 23 and ascending and maximum phase of current Solar Cycle 24. The dataset available online and downloaded from http://www.sidc.be/silso/datafiles. The Sunspot Index and Long Term Solar Observatory (SILSO) is the World Data Center for the production, preservation and dissemination of the international sunspot number.

3. THEORETICAL BACKGROUND

3.1 Wavelet Transforms

Wavelet transform is an ideal technique for the analysis of real world signals that contain sharp changes and localized discontinuities [36]. The wavelet transform use different window sizes, which are able to compress and stretch wavelets in different scales or widths; these are then used to decompose a time series [37] and decompose a one-dimensional signal into two-dimensional time–frequency domains at the same time [38]. Wavelet transform can be performed using two approaches: Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). The CWT introduces a very redundant and finely detailed description of time series in terms of both time and frequency. It is particularly helpful in resolving problems involving signal detection and identification of hidden transients such as hard to detect, short – lived elements of a time series. The scales and locations used in DWT are normally based on a dyadic arrangement (i.e. integer powers of two) [39]. DWT is especially useful for time series containing sharp jumps or shifts [40].

3.2 Time Series Decomposition via the Discrete Wavelet Transform (DWT)

The DWT is usually based on the dyadic calculation of position and scale of a signal [39]. The DWT is excellent for denoising the signals [41]. The DWT of a vector is the outcome of a linear transformation resulting in a new vector that has equal dimensions to those of the initial vector [42]. The discretization of wavelet functions is accomplished using a logarithmic uniform spacing that has a coarser resolution at higher scales [43,44]. In this study all the time series (for the Solar Cycles 23 and ascending phase of current Cycle 24) were decomposed using the Duabechies (db2) and Coifman (coif5) wavelets. Daubechies and Coifman (coif5) wavelets provide compact support, which indicate that the wavelets have non-zero basis functions over a finite interval, as well as full scale and translational orthonormality properties [45,46]. These features are very important for localizing events in the time-dependent signals [46].

3.3 Wavelet Based Multifractal Formalism

The wavelet transform not only locates isolated anomalous events, but can also characterize more complex multifractal sunspot data having non isolated singularities. Multifractal objects cannot be completely described using a single fractal dimension (mono fractals). They have an infinite number of dimension measures associated with them. The wavelet transform takes advantage of multifractal self-similarities, in order to compute the distribution of their singularities. This singularity spectrum is used to analyze multifractal properties. The concepts of fractals and multifractals and their relevance to the real world data were introduced by Mandelbrot [47]. The time series of sunspot numbers usually depict fractal or multifractal features. Time series are commonly called self-affine functions as their graphs are self-affine sets that are similar to themselves when transformed by anisotropic dilations.

Mathematically, if $f(x)$ is a self- affine function representing the sunspot number then

For $x_0 \in R, \exists\, H \in R$ such that for any $\lambda > 0$,

$$f(x_0 + \lambda x) - f(x_0) \cong \lambda^H\big(f(x_0 + x) - f(x_0)\big) \quad (1)$$

Here exponent H is known as roughness or Hurst exponent. Note that if H < 1, then f is not differentiable and smaller the exponent H, the more singular is f.

For sunspot number $f(x)$, a function $h(x)$, the Holder function of f, which measures the regularity of f at which point t is associated.

The point wise Holder h of f at point x_0 is defined as:

$$h(x_0) = \lim\ sup\{h: \exists\, c > 0,\ |f(x) - f(x_0)| \leq |x - x_0|^h, |x - x_0| < \rho\} \quad (2)$$

(Here h is an integer and f is non- differential).

One may also define local exponent $h_l(x_0)$ as:

$$h_l(x_0) = \lim\ \sup\{h: \exists\, c > 0, |f(x) - f(y)| \leq c|x - y|^h, |x - x_0| < \rho, |y - x_0| < \rho\} \quad (3)$$

Where h and h_l are different in general. For example for

$$f(x) = |x|^h sin\frac{1}{|x|^\beta}, h\,(0) = 0, while\ h_l(0) = \frac{h}{1+\beta} \quad (4)$$

They have quite different properties. For instance h_l is stable through differentiation $(h_l(f', x_0) = h_l(f, x_0) - 1)$, whereas h is not. The smaller $h(x)$ is, the more irregular the function f is at t.

3.4 Partition Function

Calculation of Pointwise Lipschitz (Holder) regularity of multifractal is not possible because its singularities are not isolated and the finite numerical resolution sufficient to discriminate them. To overcome this difficulty Muzy [48] have introduced the concept of wavelet transform modulus maxima using a global partition function [49]. Let Ψ be a wavelet with n vanishing moments. Mallat [43] has shown that if f has a pointwise Holder (Lipschitz) regularity $\alpha_0 \leq n$ at v then the wavelet transform $T_\Psi f(a,b)$ has a sequence of modulus maxima that converges towards v at fine scales. The set of maxima at the scale a can thus be interpreted as a covering of the singular support of f with wavelets of scale a. At these maxima locations

$$|T_\Psi f(a,b)| \approx a^{\alpha_0+1/2} \tag{5}$$

Let $\{u_p(a)\}_{p\in z}$ be the position of all local maxima of $|T_\Psi f(a,b)| \approx a^{\alpha_0+1/2}$ at a fixed scale a. The partition function Z measures the sum at a power q of all these wavelet modulus maxima:

$$Z(q,a) = \sum_p |T_\Psi f(a,u_p)|^q \tag{6}$$

For each $q \in R$, the scaling exponent $\tau(q)$ measures the asymptotic decay of $Z(q,a)$ at fine scale a:

$$\tau(q) = \lim\inf \frac{\log Z(q,a)}{\log a} \tag{7}$$

This typically means that $Z(q,a) \sim a^{\tau(q)}$.

4. RESULTS AND ANALYSIS

In this study we have analyzed the multifractal characteristics of sunspot number during the Solar Cycle 23 and current Cycle 24 using wavelet based multifractal techniques. Sunspot numbers (SNs) are widely and frequently used in astronomy to reflect long term variations of solar activity, which has served as the primary time series to define solar activity [50-53].

4.1 Wavelet Analysis of Sunspot Numbers

In this section DWT of the sunspot numbers for Solar Cycle 23 and 24 were carried out in terms of approximations and details coefficients using Daubechies and Coifmann mother wavelets. These wavelets have fractal structure and include both highly localized wavelets and highly smooth wavelets [54]. The sunspot time series (s) is decomposed into two orthonormal components, frequency (details) and approximations component. The approximations represent the long term of data which is almost identified to original time series and the detail coefficients represent the short period fluctuations in given period range. The result of analysis was shown in Figs. 1 – 4. In figures, the X-axis represents the time period of the sunspot time series. In all figures first panel represents the variation in sunspot number time series. The second panel gives the approximation coefficient "a_4" of all the figures. It separates the short term anomalous variation from the long term variations.

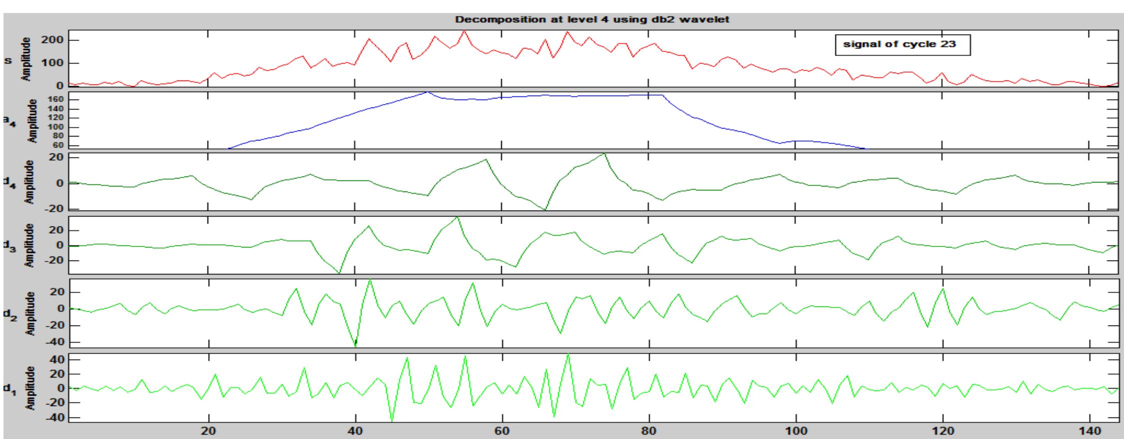

Fig. 1. DWT decompositions of Solar Cycle 23 using Daubechies 2 wavelet. Anomalous variations found between 45-60 no. of samples (i.e. at the maximum phase of cycle)

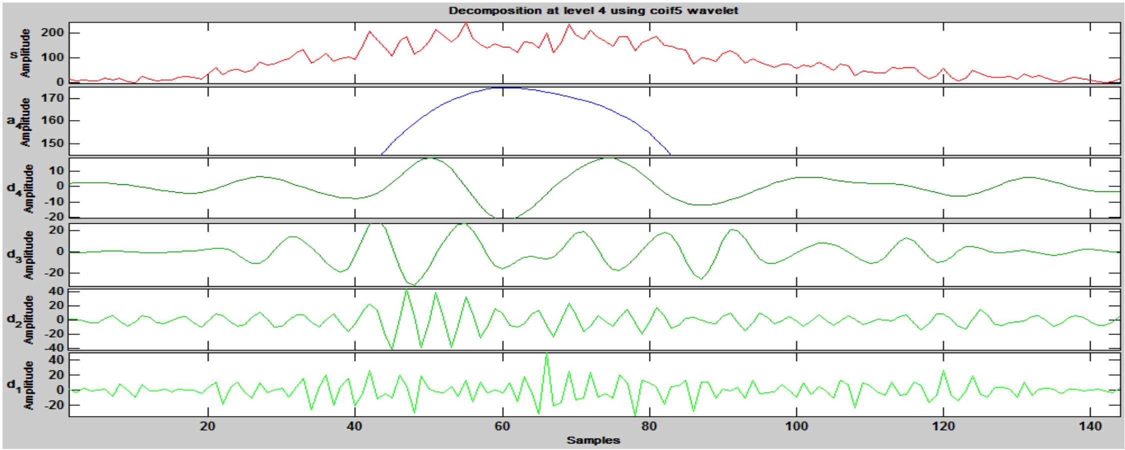

Fig. 2. DWT decompositions of Solar Cycle 23 using Coifman 5 wavelet. Anomalous sharp variations found between 40-60 no. of samples (i.e. at the maximum phase of cycle)

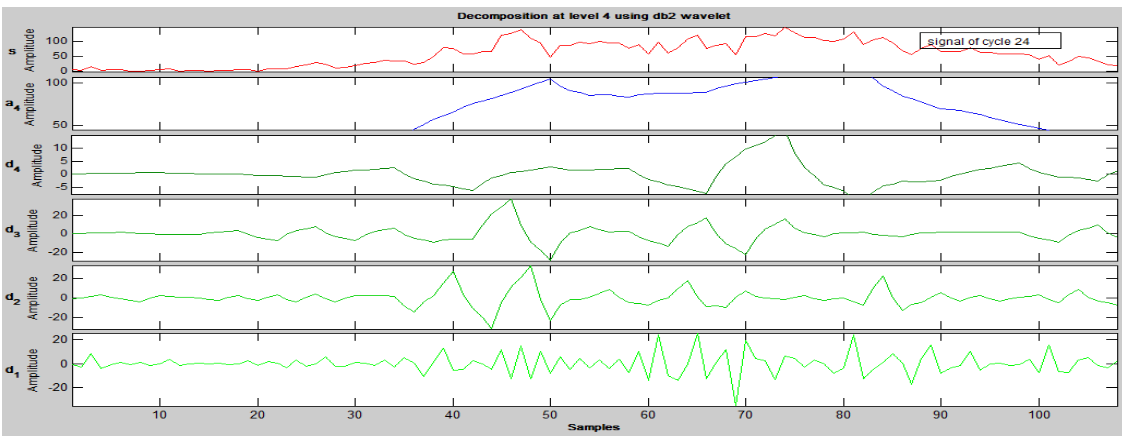

Fig. 3. DWT decompositions of Solar Cycle 24 using Daubechies 2 wavelet. Anomalous variations showing between 40 to 50 no. of samples (i.e. at the maximum phase of cycle)

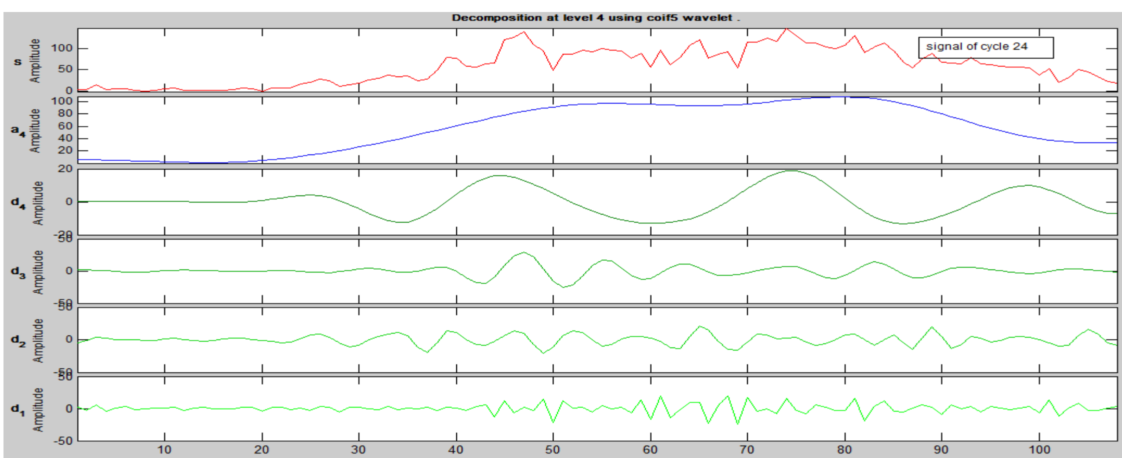

Fig. 4. DWT decompositions of Solar Cycle 24 using Coifman 5 wavelet. Anomalous sharp variations found between 40 to 50 no. of samples (i.e. at the maximum phase of cycle)

On the other hand, other four parts d_1,d_2, d_3 and d_4 represent detail coefficients of the sunspot time series. The detail coefficients reveal that the field strength changes between positive and negative values. This indicates the existence of a strong and variable magnetic field on the Sun as sunspots. The detail coefficients of sunspot numbers show that high frequency components during the initial and main phase of the solar cycle and their time evolution. It was noticed that very high frequency components present only during the main phase of the respective solar cycles and they are very strong in amplitude and stable for higher level of decomposition. Even though there are some strong fluctuations in the main phase and recovery phase but they are not as persistent as that present during the initial phase and main phase of the cycles.

4.2 Multifractal Analysis of Sunspot Numbers

In this section, we have done multifractal analysis of sunspot numbers time series [26,27].

The Legendre spectrum was calculated by FRACLAB software, developed at INRIA and available online at http://www-rocq.inria.fr. Since the sunspot data have some missing values we take the longest segments of sunspot data without any missing values. The order of the magnitude of the length of each segment is about 120, thus permitting to obtain reliable estimates of the singularity spectrum and multifractal parameters. Figs. 5 and 7 (Right panel) shows the Legendre spectra for the selected segments for both solar cycle. All the spectra present the typical single-humped shape that characterizes multifractal nature of sunspot number. The spectra of the segments for each sunspot time series, calculated for different time intervals are not identical. Nonlinear fluctuations are possibly due to the presence of multifractal processes. The smaller values of α correspond to the burst of events, while higher values of α correspond to events occurring sparsely [55]. The spectrum gives geometrical information pertaining to the dimension of sets of points in a signal having a given Holder exponent. This is the most precise spectrum from a mathematical point of view, but is also difficult one to estimate. Large deviation spectrum yields statistical information, related to the probability of finding a point with a given Holder exponent in the signal. More precisely, it measures how this probability behaves with the change in resolution.

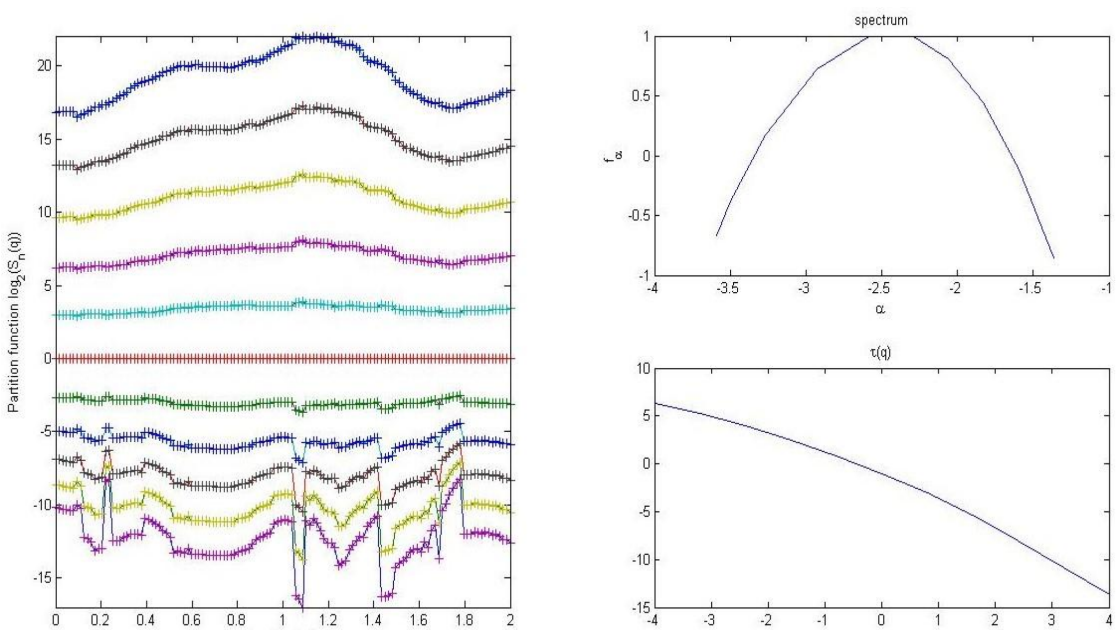

Fig. 5. Multifractal analysis of Solar Cycle 23 using CWT (Morlet wavelet of size 8 and 128 voices, LS regression and local maxima). In figure Left panel show the Legendre spectra and Right (upper) panel gives the singularity spectrum and Right (lower) panel for scaling function of the considered time series

Legendre spectrum is a concave approximation to the large deviation spectrum. Its main purpose is to yield much more robust estimates, though at the expense of a loss of information. It could base on box method or CWT techniques. In the sequel we show some sample results for the spectra computed with the Legendre technique. Figs. 6 and 8 show the results of the CWT (Morlet wavelet) based estimation of the Legendre spectrum which represents an approximation of the spectrum for two different Solar Cycles. Each figures consist of two parts in which the first part on the top of each figure represents the signal or raw data. The second part of figure shows the analyzed pattern with the application of Morlet wavelet of size 8 and 128 voices of different Solar Cycles. The non-parametric point wise Holder regularity approach based on CWT with Morlet wavelet for the Solar Cycle 23 and ascending phase of Cycle 24 was shown in Figs. 9 and 10. The Holder exponent is used for the study that characterizes the regularity of the measure (function) of the magnetic field strength of sunspot under consideration at either pointwise regularity or local regularity.

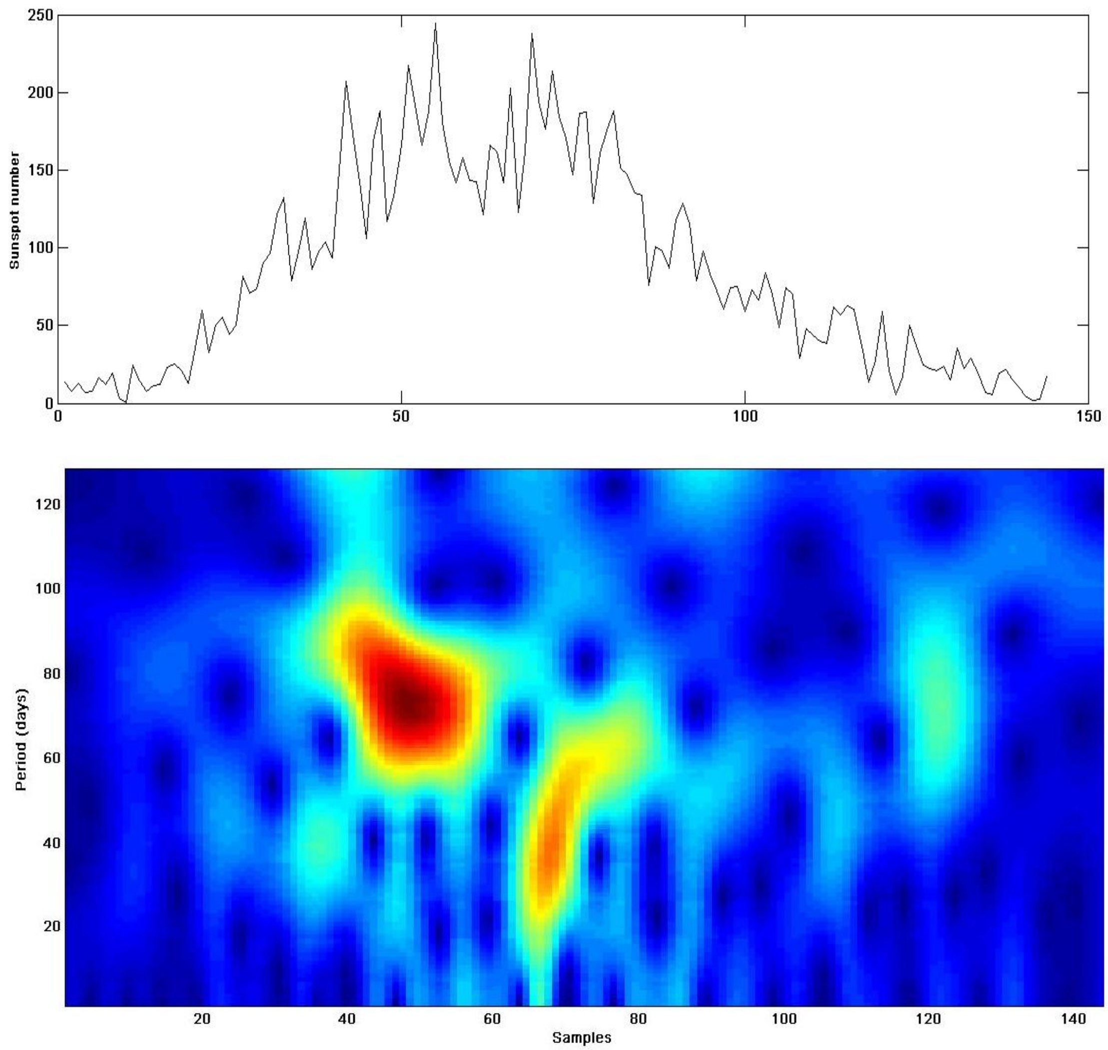

Fig. 6. CWT of Solar Cycle 23 using a Morlet wavelet of size 8 and 128 voices. In figure X axis represents the number of samples and Y axis represents scale. The upper panel shows the raw data of the time series and lower panel its continuous wavelet transform (CWT)

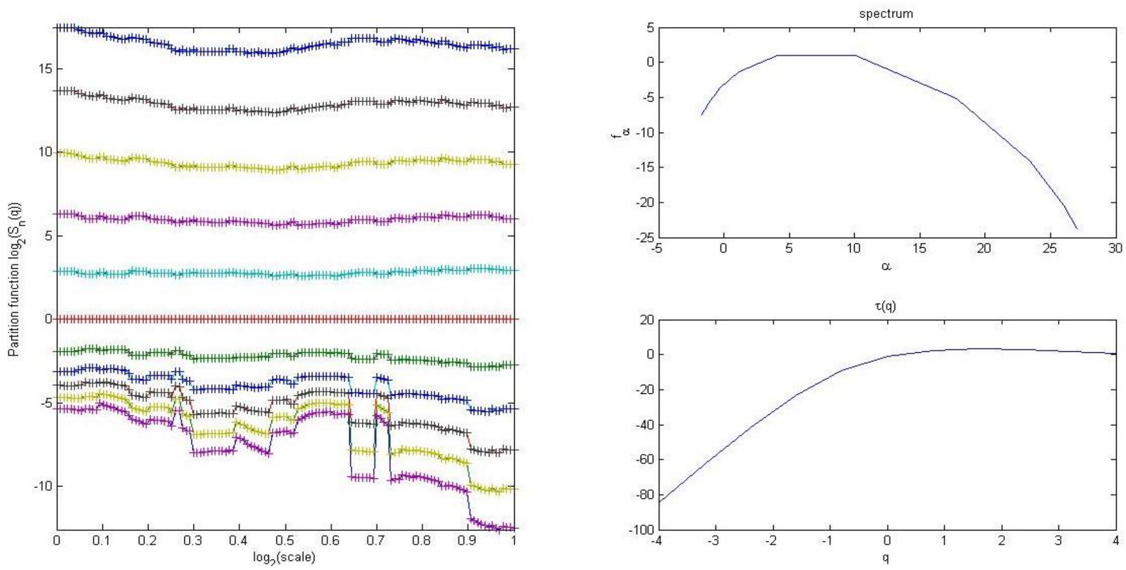

Fig. 7. Multifractal analysis of Solar Cycle 24 using CWT (Morlet wavelet of size 8 and 128 voices, LS regression and local maxima). In figure Left panel show the Legendre spectra and Right (upper) panel gives the singularity spectrum and Right (lower) panel for scaling function of the considered time series

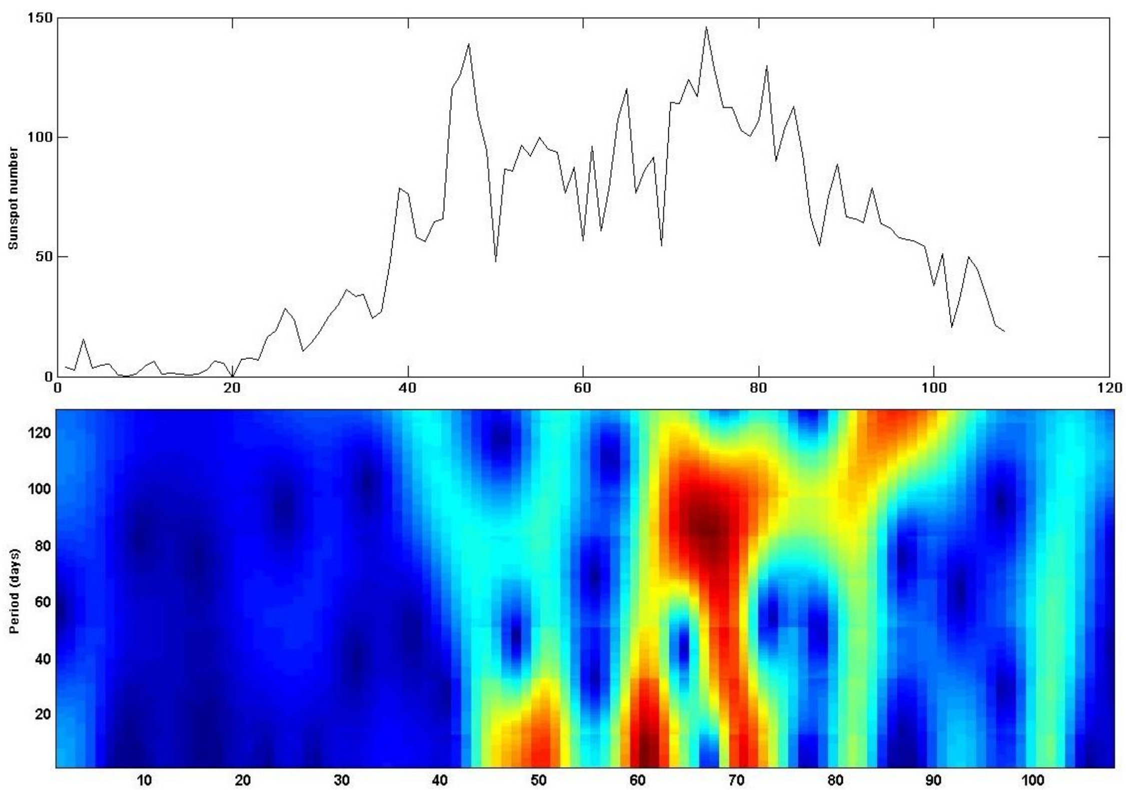

Fig. 8. CWT of Solar Cycle 24 using a Morlet Wavelet of size 8 and 128 voices. In figure X axis represents the number of samples and Y axis represents scale. The upper panel shows the raw data of the time series and lower panel its continuous wavelet transform (CWT)

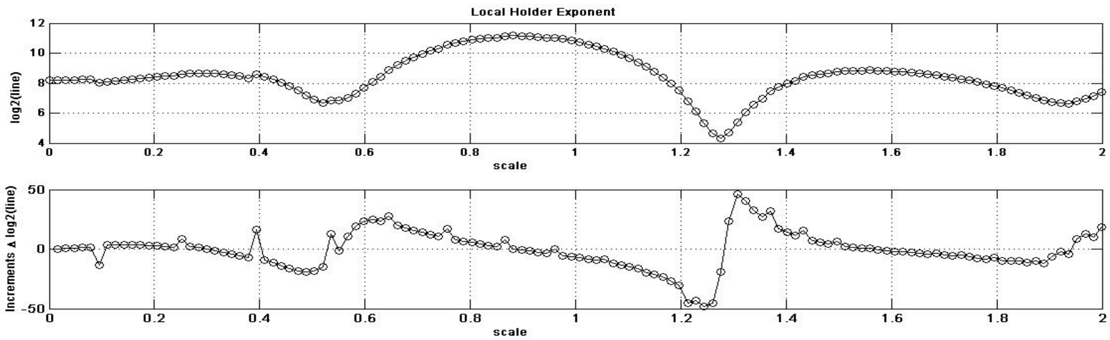

Fig. 9. Non-parametric point wise Holder regularity estimation using CWT with Morlet wavelet for the Solar Cycle 23

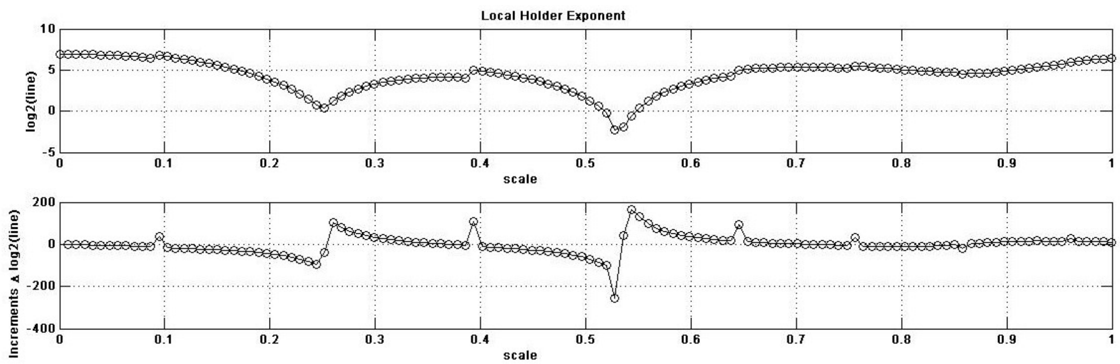

Fig. 10. Non-parametric point wise Holder regularity estimation using CWT with Morlet wavelet for the Solar Cycle 24

5. CONCLUSION

Wavelet based multifractal analysis is a useful way to characterize the multifractality in time series. The result of our analysis shows that sunspot time series reveal multifractal character during Solar Cycle 23 and ascending phase of Solar Cycle 24. The DWT (Figs. 1 - 4) of Sunspot time series shows the phenomenological difference between Solar Cycle 23 and Cycle 24. It present Solar Cycle 23 has large range fluctuation due to the contribution of highly magnitude solar activities whereas during the Cycle 24 it is not noticed hence it is a quite Cycle except during the maximum phase revels the large fluctuations. The wavelet spectrum depicts in Figs. 6 and 8 reveals the time at which large variations in time series occur, which is more important for the investigation of multifractality. Various phenomenon related to Sun such as solar flares, sunspot, coronal mass ejection (CME) etc. Three main multifractal spectra viz. the Housedroff, large deviation and Legendre spectra (see Figs. 5 and 7) provides information as to which singularities occur in sunspot time series and which are dominant. Multifractal analysis provided both a local and a global description of the singularities of a signal. Recent studies have shown that non-Gaussian fluctuation is responsible for the presence of extreme events in space plasmas. Recently, using a non-extensive approach Balasis et al. [56] suggested emergence of two distinct phases: (i) the phase where the intense magnetic storms cause a higher degree of magnetic field organization, and (ii) the phase which characterizes the normal periods with lower magnetic field coherence. The phase (i) may be associated with the presence of different kinds of large scale coherent structures, also pointed out by Chang et al. [57]. The wavelet spectrum displayed in Figs. 6 and 8 reveals the time at which large variations occur which is very important in the investigation of multifractality. Thus, we conclude that Solar Cycle 23 commonly shows the multiracial nature around the initial maximum and declining phase of cycle rather than Solar Cycle 24. It was not effective during the initial phase of cycle. Solar Cycle 24 has more fractal nature at the maximum phase and we wait for more result till complete the ongoing cycle.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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